Closing Fri: 3.10
Closing Mon: 4.1(1) and 4.1(2)
Closing Wed: 4.3

3.10 Linear Approximation

Idea: "Near" the point (a,f(a)) the graphs of $y = f(x)$ and the tangent line

$$y = f'(a)(x - a) + f(a)$$

are very close together.

We say the tangent line is a **linear approximation** or **linearization** or **tangent line approximation** to the function. Sometimes it is written as

$$L(x) = f'(a)(x - a) + f(a)$$

In other words:
If $x \approx a$, then

$$f(x) \approx f'(a)(x - a) + f(a)$$

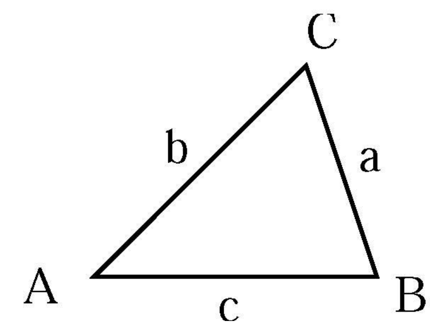
Examples:

1. Find the linear approximation of $f(x) = \sqrt{x}$ at $x = 81$. Then use it to approximate the value of $\sqrt{82.}$

2. Find the linearization of $g(x) = \sin(x)$ at $x = 0$. Then use it to approximate the value of sin(0.03).

3. Using tangent line approximation estimate the value of $\sqrt[3]{8.5}$.

Some Homework Hints:



Problem 10: Suppose that *a* and *b* are pieces of metal of constant length which are hinged at *C* (note: length c can change).

At first: $A = \pi/4$ rad (45^o) and

$B = \pi/3$ rad (60^o).

You then widen *A* to 46^o ($46\pi/180$ rad)

The problem asks you to *use the tangent line approximation to estimate new the angle B. That is*, <u>find the tangent line</u>

B = ??? (A - ???) + ???

There is a big hint that says the ``law of sines'' gives:

$$\frac{b}{a} = \frac{\sin(B)}{\sin(A)}$$

Problem 8: Total surface area of a cone shaped solid:

$$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$$

At first: h = 8 cm, r = 6 cm
You then increase the height by 0.26 cm. (to 8.26 cm)

You want to change the dimensions in such a way that *the total surface area remains constant*.

The problem asks you to *use the tangent line approximation to estimate the new value of r. That is,* <u>find the tangent line</u>

r = ??? (h - ???) + ???

Ch. 4 Derivative Applications

4.1: Max/Min Values

Terminology for analyzing a function:

Let y = f(x).

1. What is the domain?

   We will only talk about points that are already in the domain.

2. What are the endpoints?

   Is the problem restricted to a closed interval?

3. What are the “critical numbers”?

   A **critical number** is a location, x = a, where either

   (a) f’(a) = 0, or

   (b) f’(a) does not exist

4. Are there peaks and valleys?
(also known as *extrema* or *optima*)

(a) A **local maximum** is higher than *nearby* points on both sides.

(b) A **local maximum** is lower than *nearby* points on both sides.

(c) The **absolute max** (or global max) is the highest overall point.

(d) The **absolute min** (or global min) is the lowest overall point.

Small Technical Note:
The **value** of a function, y = f(x), is the output y-value.
A question asking you to find the absolute max of a function is asking for the y-value. The x is where the max would *occur*, not the max value.

Key observations:

1. Local max/min always occur at critical numbers! (Fermat's Thm)

2. Absolute max/min always occur at critical numbers or endpoints! (Extreme Value Theorem)

Thus, in all max/min problems our first steps will be to find the critical numbers.